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ORIGINAL CONTRIBUTION

STUDIES ON SOME THERMODYNAMIC PROPERTIES OF QGP AT EXTREMELY HIGH TEMPERATURE USING QUADRATIC GUP

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ABSTRACT

Many theories of quantum gravity such as string theory, doubly special relativity etc. predict the existence of a minimum observable length at the Planck scale, which leads to the modification of the celebrated Heisenberg Uncertainty Principle(HUP) to the Generalized Uncertainty Principle (GUP). Quadratic GUP will be implemented here in deriving some thermodynamic properties of quark-gluon-plasma (QGP) at extremely high temperature (at Planck scale) using MIT bag model.

KEYWORDS: Generalized Uncertainty Principle (GUP), Bag pressure, Quark Fermi momentum, Critical baryon density

1. INTRODUCTION

Physical theories with GUP have attracted huge attention over the years. This idea got immense support from the concept of minimum length arose from string theory since string theory cannot probe a length smaller than string length [1]. Despite the immense support from the string theory also got considerable impetus from the loop quantum gravity. So the concept of minimum length is a universally accepted proposition at the present time. Theoretically minimum length can be introduced by the modified uncertainty relation $\Delta x \Delta p \geq \frac{\hbar}{2} f(\beta p)$.

For $f(\beta p) = 1$, the modified uncertainty relation reduces to usual Heisenberg uncertainty principle. The function $f(\beta p)$ can be chosen at our will, however the parameter β has to be constrained from the experimental observation. Different type of $f(\beta p)$ has been found to exercise on different physical system [2-9]. The most important generalization and which has direct link with string theory is the quadratic GUP that arises from function $f(\beta p) = [1 + \beta(\Delta p)^2]$, Where β is called GUP parameter and has origin in nature of space-time. It is defined as $\beta = \frac{\beta_o}{M_{Pl} C^2} = \frac{\beta_o {l_{pl}}^2}{\hbar^2}$, β_o is a positive dimensionless parameter and it is normally assumed that β_o not far from unity. $M_{pl}C^2 \sim 10^{19}$ GeV and l_{pl} is Planck length is of the order of 1.616×10^{-35} m. The second term on the RHS of $f(\beta p)$ is important at very high energies/small length scales (i.e., $\Delta x \sim l_{pl}$). The introduction of GUP automatically includes the Planck scale correction into the thermodynamic properties of a physical system that makes the GUP proposition more interesting.

Consider the case of a quark-gluon system in thermal equilibrium at high temperature T, within a volume V. Here quarks and gluons are idealized to noninteracting and massless and there is no net baryon number. The number of quarks and the number of anti-quarks in the system is equal. QGP is a state of matter where

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nucleon loses their complete identity and melts into a soup of quark and gluon. It may occur at very high energies (temperature) and / or with very high densities. Although there is no concrete experimental evidence regarding QGP state, but it is believed that it is possible to create QGP in heavy ion collision. It is also believed that there is some indirect signature after Big-Bang during the evolution of the Universe matter had to cross that state. At that time density as well as energy was too high and the quantum effect of gravity was not ignorable which can be taken into consideration with the modified uncertainty algebra as it is concomitant with string theory early stage of evolution of the Universe. Since the GUP apparently modifies the fundamental commutator bracket between position and momentum operators, then it is natural to expect that this would result in considerable modifications in all thermodynamic properties. In this article, we want to make an attempt to derive some thermodynamic quantities of QGP at extremely high temperature with the help of quadratic GUP up to the leading order (1^{st} order) of GUP parameter(β). Here MIT bag model is used to do so. According to bag model it is assumed that all matters are kept inside a bag having inward bag pressure B at some temperature T.

2. DESCRIPTION OF QUADRATIC GUP

To establish a concrete theory of quantum gravity is currently one of the main challenges in theoretical physics. Various approaches predict the existence of a minimum length scale [3, 4] that leads to the modification of the Heisenberg Uncertainty Principle-

$$\Delta x \Delta p \geq \frac{\hbar}{2} \tag{1}$$

And to the Generalized Uncertainty Principle (GUP) [5, 6]

$$\Delta x \Delta p \geq \frac{\hbar}{2}(1 + \beta(\Delta p)^2 + \beta < p >^2) \tag{2}$$

Equation (2) comes from the general form of quadratic GUP [7,8] given below.

$$\Delta x_i \Delta p_i \geq \frac{\hbar}{2}\ (1 + \beta((\Delta p)^2 + < p >^2) + 2\beta((\Delta p_i)^2 + < p_i >^2))) \tag{3}$$

Which follows from the modified commutation relation [8] given below.

$$[x_i, p_j] = i\hbar(\delta_{ij} + \beta(p^2\delta_{ij} + 2p_i p_j)) \tag{4}$$

The commutation relation (4) suggests the existence of minimal observable length as minimum uncertainty in position, $(\Delta x)_{min} = \hbar\sqrt{\beta}$ and admits the following representation in position space [10, 11] $x_i = x_{0i}$ and $p_i = p_{0i}(1 + \beta p_0^2)$.

Where x_{0i} and p_{0j} satisfy the canonical commutation relation $[x_{0i}, p_{0j}] = i\hbar\delta_{ij}$. Here x_{0i} and p_{0i} can be interpreted as position and momentum low energies (having standard representation in position space i.e., $x_{0i} = x_{0i}$ and $p_{0i} = -i\hbar\frac{\partial}{\partial x_{oi}}$) and x_i and p_i as that at high energies.

The change in Heisenberg algebra has a direct effect on the phase space volume. In the following section we will discuss how phase space volume gets deformed for this GUP.

2.1. Phase-space volume up to the leading order in β:

To make necessary correction in the thermodynamic properties of QGP due to this GUP, one need to modify phase space volume and consequently the number of quantum states per momentum space volume (density of states) changes accordingly. It is known that for different GUP, the density of state gets modified in different manner. For instance in the article [12], It has been shown that for the quadratic GUP, where $f(p) = 1 + \beta p^2$, the phase space volume for any arbitrary dimension D becomes[12]-

$$\frac{V d^D p}{(1+\beta p^2)^D} \tag{5}$$

Where V is the coordinate space volume. And correspondingly density of states becomes-

$$dN = \frac{V d^D p}{(2\pi\hbar)^D (1+\beta p^2)^D} \tag{6}$$

In this article we put $\hbar = c = 1$ (in natural unit). So, density of states becomes-

$$dN = \frac{V d^D p}{(2\pi)^D (1+\beta p^2)^D} \tag{7}$$

The general uncertainty relation we have considered is a general one, however the first order (of β) term is significant and that only have been considered in all the computation of modified phase space volume [12-14]. In this article, some thermodynamic properties of QGP will be extended according to the modified phase space volume.

3. CALCULATION OF THERMODYNAMIC QUANTITIES OF QGP

The number of quarks in a volume V, within the momentum interval p to $(p + dp)$ will be the number of states in a volume V and within the same interval of momentum, is given by (taking $\hbar = 1, c = 1$) i.e., in natural units.

$$dN_q = \frac{g_q V}{(2\pi)^3} 4\pi p^2 dp \tag{8}$$

In order to see the modified number of states in volume V within the momentum range between p and $(p + dp)$ we need to consider the phase space volume concomitant to the generalized uncertainty relation $[x, p] = i\hbar(1 + \beta p^2)$. since all the states are occupied by quarks, then from equation (6), then number of states becomes-

$$dN_q = \frac{g_q V}{(2\pi)^3} 4\pi p^2 dp \frac{1}{(1+\beta p^2)^3} \tag{9}$$

Where g_q is degeneracy of the quarks, explicitly $g_q = N_c N_s N_f$. $N_c = 3(\text{number of colors})$, $N_s = 2(\text{number of spin states})$ and $N_f = 2(\text{if heavy quarks are not considered})$.

And the total number of quark in volume V will acquire the following expression in the present case:

$$N_q = \frac{g_q V}{(2\pi)^3}\int_0^\infty \frac{4\pi p^2 dp}{(1+\beta p^2)^3\left(1+e^{\frac{p}{T}}\right)} = \frac{g_q V}{2\pi^2}\int_0^\infty \frac{p^2 dp}{(1+\beta p^2)^2\left(1+e^{\frac{p}{T}}\right)} \quad (10)$$

As $\beta \ll 1$, retaining the term up to first order of β we have by integrating

$$N_q = N_{\bar{q}} = 0.09\, g_q V T^3(1 - 36.23\beta T^2) \quad (11)$$

So the number density of the quark gas in the present situation reads

$$n_q = \frac{N_q}{V} = 0.09 g_q T^3(1 - 36.23\beta T^2) \quad (12)$$

The energy of the massless quarks, using modified phase space volume, in a volume V is

$$E_q = \frac{g_q V}{(2\pi)^3}\int_0^\infty \frac{4\pi p^3 dp}{(1+\beta p^2)^3\left(1+e^{\frac{p}{T}}\right)} = \frac{g_q V}{2\pi^2}\int_0^\infty \frac{p^3 dp}{(1+\beta p^2)^3\left(1+e^{\frac{p}{T}}\right)}$$

Again, since $\beta \ll 1$, retaining the term up to first order of β we have by integrating

$$E_q = \frac{7}{8} g_q V \frac{\pi^2}{30} T^4(1 - 6.32\beta\pi^2 T^2) \quad (13)$$

Consequently the energy density of the quark gas reads

$$\epsilon_q = \frac{E_q}{V} = \frac{7}{8} g_q \frac{\pi^2}{30} T^4(1 - 6.32\beta\pi^2 T^2) \quad (14)$$

From the relation between the pressure (P_q) and energy density (ϵ_q) we have

$$P_q = P_{\bar{q}} = \frac{E}{3V} = \frac{1}{3}\left[\frac{7}{8} g_q \frac{\pi^2}{30} T^4(1 - 6.32\beta\pi^2 T^2)\right] \quad (15)$$

Therefore entropy per quark or anti-quarks, under a constant pressure is

$$S_q = S_{\bar{q}} = 1.4\, \frac{(1-41.62\beta T^2)}{(1-36.23\beta T^2)} \quad (16)$$

Let us now proceed to calculate now the energy of the gluons within volume V and temperature T. Since gluons having degeneracy g_g obey Bose-Einstein statistics, so the energy expression for gluon will be

$$E_g = \frac{g_g V}{2\pi^2}\int_0^\infty \frac{p^3 dp}{\left(e^{p/T}-1\right)} \quad (17)$$

With quadratic GUP the above equation takes the following generalized form.

$$E_g = \frac{g_g V}{2\pi^2}\int_0^\infty \frac{p^3 dp}{(1+\beta p^2)^3\left(e^{p/T}-1\right)} \quad (18)$$

$$E_g = \frac{g_g V}{30}\pi^2 T^4(1 - 56.25\beta T^2) \quad (19)$$

Therefore the Pressure due to gluons at temperature T is given by using pressure energy relation for ideal gas $P_g = \frac{E}{3V}$

$$P_g = \frac{g_g}{90}\pi^2 T^4(1 - 56.25\beta T^2) \quad (20)$$

The number density of gluons in volume V at temperature T is

$$n_g = \frac{g_g}{2\pi^2}\int_0^\infty \frac{p^2 dp}{(1+\beta p^2)^2\left(e^{p/T}-1\right)}$$

$$n_g = \ 0.12 g_g T^3(1-20.7\beta T^2) \tag{21}$$

Therefore entropy per gluon is

$$S_g = 1.2\ \frac{(1-56.25\beta T^2)}{(1-20.7\beta T^2)} \tag{22}$$

Now, pressure of quark-gluon plasma (QGP) is obtained as

$$P_{QGP} = P_q + P_{\bar{q}} + P_g = \ 2\frac{1}{3}\left[\frac{7}{8} g_q \frac{\pi^2}{30} T^4(1-6.32\beta\pi^2 T^2)\right] + \frac{g_g}{90}\pi^2 T^4(1-56.25\beta T^2) \tag{23}$$

Factor 2 comes due to contribution of quark and anti-quark.

4. RESULTS AND DISCUSSION

On the basis of quadratic GUP due to modified commutation relation $[x,p] = i\hbar(1+\beta p^2)$ at Planck scale, some thermodynamic properties of QGP at high temperature are derived in terms of β. All the above results are deduced up to the leading (first) order of GUP parameter (β). It is seen from the expression of energy, pressure and entropy that these quantities depend on temperature and GUP parameter.

If GUP parameter (β) be taken as zero, eqn.(16) and (22) gives the usual results. Since with increase in temperature, entropy of everything increases.So, to hold good result GUP parameter (β) must be taken as negative quantity for eqn.(16) and (22).

If $\beta = 0$, eqn.(23) gives the following expression of QGP pressure

$$P_{QGP} = \ 37\frac{\pi^2}{90}T^4 \tag{24}$$

Here quark of $N_f = 2$, $g_g = 16$ and $g_q = 12$ are considered.

The critical temperature, at which the quark-gluon pressure is equal to the bag pressure B, is given by

$$T_c = \left(\frac{90}{37\pi^2}\right)^{1/4} B^{1/4} \tag{25}$$

Similarly, for a particular value of β and B, we have different critical temperature from eqn.(23). If the quark matter in a bag is heated up to a high temperature greater than the critical temperature, the quark matter inside the bag must have a pressure which is greater than that of the bag pressure. When this happen, the bag will not be able to hold the quark matter inside the bag and the quark matter will be deconfined. The deconfined phase of quark matter is given the general name 'quark-gluon plasma'. Here β

plays a significant role in determining critical temperature under the said framework.

5. CONCLUSION

In this article, quadratic GUP have been used to describe some properties of QGP up to the leading order of GUP parameter (β) using MIT bag madel. It is seen in above calculations that β plays a significant role in determining critical temperature as well as entropy per quark and gluon. Since it is assumed that β is a factor that comes due to quantum gravitational correction, so it may say that it is an attempt to incorporate correction due to quantum gravity in an indirect manner to strengthen the theories at the vicinity of Planck scale.

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