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ORIGINAL CONTRIBUTION

Analysis of predator-prey model having Holling type-II functional response with disease in both species

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ABSTRACT

A system of non-linear differential equations is proposed as a predator-prey mathematical model with disease in both prey and predator. Total population is divided into four groups, namely susceptible prey, infected prey, susceptible predatorand infected predator. The dynamical behavior of this system about each of the equilibrium points has been observed. The system locally asymptotically stable at the interior equilibrium point is observed. It has been also observed that a Hopf- bifurcation can happen about the interior equilibrium point when the rate of infection crosses some parameter threshold. The analytical findings are supported by the numerical observations.

Key Words: Predator-prey, Infection, Disease transmission, Stability,Hopf-bifurcation

1. INTRODUCTION

In recent times, the effects of infectious disease is an important regulating factor for human and animal population size. Authors [1, 2, 3, 4] are studied several years past the dynamics of predator-prey systems with prey populations affected by some transmissible disease. From these outcomes different mathematical models have been established. In eco-epidemiological systems with disease, the predator takes a disproportionately high number of parasite infected prey is observed [5, 6]. In the publications [7, 8, 9], they are analyzed a predator-prey model with logistic growth in prey species affected by SIS parasitic infection where the infected prey is more vulnerable to predation. It has been observed that earlier research works are mostly focused on parasite infection in prey pop- ulations only. The Researchers considered some predator-prey model in which infection of predator through eating prey[2, 3, 10] or spread of infection in predators [11, 12, 13, 14]. The authors in the publications [15, 16] have analyzed a predator-prey model in which disease in both populations. The Researchers [17, 18, 19] have considered Holling type-II intake rate function where the disease is not genetic and no immunity.

In this work, a predator-prey model with disease in both populations is proposed. The growth in susceptible prey population is only considered and it follows the logistic law. The predation rates are different for the susceptible predator and the infected predator. Consider the infection is not transmitted vertically in the predator. In the reference [21] in which the author considers phytoplankton (as for example Cryptophytic) and zooplankton (as for example Rotifers) are both infected by some viral disease in marine planktonicsystems. To explain the scenario, a mathematical model is formulated in which the viral disease could infect both the predator and prey populations, but it cannot be transmittedbetween them.

**Corresponding Address:*

2. MATHEMATICAL MODEL

To construct the model, we make the following assumptions:

- In the absence of disease the prey population density (N) grows following the logisticlaw:

$$\frac{dN}{dt} = rN(1 - N/k)$$

where k ($k \in R+$) is carrying capacity and r($r \in R+$) is intrinsic birth rate.

- In the presence of disease, at any time t such that, $N = X + Y$ and $P = Z + W$, where

P =Predator population density, X=susceptible prey, Y =infected prey,

Z = susceptible predator, W =infected predator.

Here, only the susceptible prey population follows the logistic law:

$$\frac{dX}{dt} = rX\{1 - (X + Y)/k\}$$

- Under the mass action law, the susceptible prey X becomes infected at the rateof infection α and also the susceptible predator Z is infected at the rate of infection β. Infected prey and infected predators do not recover from the disease is assumed.

- The infected predators are not fit to be able to catch a susceptible prey and a susceptible prey is caught by a susceptible predator only. But an infected prey, being weak and more vulnerable, is available for predation by both susceptible and infected predators.

- Susceptible predator consume susceptible prey by $c_1XZ/(1 + aX)$ and infected prey by c_2YZ at constant rates c_1 and c_2 respectively but infected predator consume only infected prey by c_2eYZ at constant rate c_2e, where e ($0 < e < 1$) is the impact of disease on thepredation rate and a is half saturation constant in prey.

From above basic assumptions the mathematical model is

$$\frac{dX}{dt} = rX\left\{1 - \frac{X+Y}{k}\right\} - \alpha XY - c_1\frac{XZ}{(1+aX)} \equiv F_1(X,Y,Z,W),$$

$$\frac{dY}{dt} = \alpha XY - r_1Y(X+Y) - c_2Y(Z+eW) - d_1Y \equiv F_2(X,Y,Z,W),$$

$$\frac{dZ}{dt} = c_1 e_1 \frac{XZ}{(1+aX)} + c_2 e_2 YZ - \beta ZW - d_2 Z \equiv F_3(X,Y,Z,W),$$

$$\frac{dZ}{dt} = \beta ZW - d_3 W \equiv F_4(X,Y,Z,W), \qquad (1)$$

with initial conditions,

$$X(0) = X_0 > 0, Y(0) = Y_0 > 0, Z(0) = Z_0 > 0, W(0) = W_0 > 0 \qquad (2)$$

where $e_1(0 < e_1 < 1)$, $e_2(0 < e_2 < 1)$ are the conversion rates factor for the suscepti- ble predator due to consumption of susceptible prey and infected prey respectively and d_1, d_2 , d_3 are the death rates of infected prey, susceptible predator and infected predatorrespectively.

3. BOUNDEDNESS OF THE SYSTEM

Theorem 1. Every solution of the system (1) is bounded.

Proof: Let $V(t) = X(t) + Y(t) + Z(t) + W(t)$.

Now using the equations (1), we have

$$\frac{dV}{dt} = \frac{dX}{dt} + \frac{dY}{dt} + \frac{dZ}{dt} + \frac{dW}{dt}$$

$$= rX\left(1 - \frac{X+Y}{k}\right) - c_1 \frac{XZ}{(1+aX)} - c_2 Y(Z + eW) + c_1 e_1 \frac{XZ}{(1+aX)}$$

$$+ c_2 e_2 YZ - d_1 Y - d_2 Z - d_3 W$$

$$= rX - \frac{r}{k}X^2 - c_1(1-e_1)\frac{XZ}{(1+aX)} - c_2(1-e_1)YZ$$

$$+ c_2 eYZ - d_1 Y - d_2 Z - d_3 W$$

$$\leq rX - \frac{r}{k}X^2 - d_1 Y - d_2 Z - d_3 W, \text{ if } e_i < 1, \text{ for } i = 1, 2.$$

Therefore,

$$\frac{dV}{dt} + \mu V \leq X\left(r + \mu - \frac{r}{k}X\right) - (d_1 - \mu)Y - (d_1 - \mu)Z - (d_1 - \mu)W.$$

Let $\mu = \min\{d_1, d_2, d_3\}$, then $\frac{dV}{dt} + \mu V \leq X\left(r + \mu - \frac{r}{k}X\right) \leq \frac{k(r+\mu)^2}{4r}$.

If $e_i < 1$, for $i = 1, 2$ and $\mu = \min\{d_1, d_2, d_3\}$, then for each $\mu \geq 0$ the above inequality becomes

$$\frac{dV}{dt} + \mu V \leq m, \text{ where } m = \frac{k(r+\mu)^2}{4r}$$

Now using the reference [22] we have,

$$0 \le V(t) \le \frac{m}{\mu}(1 - e^{-\mu t}) + V(0)\, e^{-\mu t}.$$

As $t \to \infty$, then $0 \le V(t) \le \frac{m}{\mu}$. Hence $V(t)$ is bounded. Thus every solution of the system is bounded.

4. EQUILIBRIA ANALYSIS

- The equilibria $E_0(0,\ 0,\ 0,\ 0)$ and $E_1(\ k,\ 0,\ 0,\ 0\ )$ exist for all parametric values.
- The predators free equilibrium point $E_2(X_2,\ Y_2,\ 0,\ 0\ )$ exists if $R_0 > 1,\ R_1 > 1$, where

$$R_0 = \frac{\alpha}{r}, R_1 = \frac{k(\alpha - r_1)}{d_1}.$$

$$X_2 = \frac{krr_1 + d_1(r + k\alpha)}{(\alpha - r_1)(r + k\alpha) + rr_1}, Y_2 = \frac{r(k\alpha - kr_1 - d_1)}{(\alpha - r_1)(r + k\alpha) + rr_1},$$

- The infected prey and infected predator free equilibrium point

$E_3(X_3, 0,\ Z_3,\ 0)$ exists if $R_2 > 1,\ R_3 > 1$ where $X_3 = \frac{d_2}{c_1e_1 - ad_2}$,

$$Z_3 = \frac{r}{kc_1}\left(1 + a\frac{d_2}{c_1e_1 - ad_2}\right)\left(1 - \frac{d_2}{k(c_1e_1 - ad_2)}\right),\ R_2 = \frac{c_1e_1}{ad_2},\ R_3 = \frac{d_2}{c_1e_1 - ad_2}.$$

- The infected prey free equilibrium point $E_4(X_4, 0, Z_4, W_4)$ if $R_4 > 1$, where

$R_4 = \frac{\beta r}{cd_3}, Z_4 = \frac{d_3}{\beta},\ W_4 = \frac{1}{\beta}\left(c_1e_1\frac{X_4}{1 + aX_4} - d_2\right)$ *and* X_4 is the positive root of the equation

$$\frac{ar}{k}\rho^2 + r\left(\frac{1}{k} - a\right)\rho + \frac{c_1d_3}{\beta} - r = 0$$

- The infected predator free equilibrium point $E_5(X_5, Y_5, Z_5, 0\ )$ exists if $R_5 > 1$, where

$$R_5 = \frac{e_2(c_2kr + c_1kd_1) + c_1e(r + k\alpha)}{e_2(c_2r + c_1k\alpha) + d_2(r + k\alpha)}, Y_5 = \frac{1}{c_2e_2}\left(d_2 - c_1e_1\frac{X_5}{1 + aX_5}\right),$$

$$Z_5 = \frac{(\alpha - r_1)X_5 - r_1Y_5 - d_1}{c_2}$$ and X_5 is the positive root of the equation

$$A_1\rho^3 + A_2\rho^2 + A_3\rho + A_4 = 0,$$

$$A_1 = \frac{r}{k}a^2c^2{}_2e_2,\ A_2 = c_2L_1a(d_2a - c_1e_1) - c^2{}_2e_2a^2r + c_1c_2e_2a(\alpha - r_1),$$

$$A_3 = c_2L_1(2ad_2 - c_1e_1) + c_1\{c_2e_2(\alpha - r_1 - ad_1) - r_1(ad_2 - c_1e_1)\} + \frac{r}{k}c^2{}_2e_2 - 2arc^2{}_2e_2,$$

$$A_4 = -c_1(d_1c_2e_2 + r_1d_2) + c_2d_2L_1 - rc^2{}_2e_2,\ L_1 = \frac{r}{k} + \alpha\ .$$

● The positive interior equilibrium point $E^*(X^*, Y^*, Z^*, W^*)$ where $Z^* = \frac{d_3}{\beta},$ *

$$W^* = \frac{1}{\beta}\left\{c_1e_1\frac{X^*Z^*}{(1+aX^*)} + c_2e_2Y^* - d_2\right\},\ \ Y^* = \frac{1}{L}\left\{\beta(\alpha - r_1)X^* - c_1c_2ee_1\frac{X^*Z^*}{(1+aX^*)} + M\right\},$$

$L = c^2{}_2e_2e + \beta r_1,\ M = ed_2c_2 - d_3c_2 - \beta d_1$ and X^* is the positive root of the equation

$$A\rho^2 + B\rho + C = 0,$$

where $A = \frac{L_1a\beta(\alpha - r_1)}{L}, B = \frac{L_1}{L}\{\beta(\alpha - r_1) + Ma - c_1c_2ee_1\} - ar + \frac{r}{k},$

$$C = \frac{c_1d_3}{\beta} - r + \frac{L_1M}{L}.$$

5. STABILITY ANALYSIS OF THE BOUNDARY EQUILIBRIA

Let J_i denotes the jacobian matrix at the equilibrium E_i, i = 0, 1, 2, 3, 4, 5.

5.1 E_0

The jacobian matrix J_0 of the equilibrium point E_0 is given by

$$J_0 = \begin{pmatrix} r & 0 & 0 & 0 \\ 0 & -d_1 & 0 & 0 \\ 0 & 0 & -d_2 & 0 \\ 0 & 0 & 0 & -d_3 \end{pmatrix}$$

The eigenvalues of J_0 are $r, -d_1, -d_2$ and d_3. So the equilibrium point E_0 is unstable because one eigenvalues is positive.

5.2 E_1

The jacobian matrix J_1 of the equilibrium point E_1 is given by

$$J_1 = \begin{pmatrix} -r & -(\alpha + \frac{r}{k}) & \frac{c_1k}{1+ak} & 0 \\ 0 & 0 & (\alpha - r_1)k - d_1 & 0 \\ 0 & 0 & c_1e_1k/(1+ak) - d_2 & 0 \\ 0 & 0 & 0 & -d_3 \end{pmatrix}$$

The eigenvalues of J_1 are $-r, -d_3, c_1e_1k/(1+ak) - d_2$ and $(\alpha - r_1)k - d_1$.

Thus the point E_1 is stable if $\frac{d_1}{(\alpha - r_1)k} < 1$ and $\frac{d_2(1+ak)}{c_1e_1k} < 1$.

5.3 E_2

The jacobian matrix J_2 of the equilibrium point E_2 is given by

$$J_2 = \begin{pmatrix} -\frac{r}{k}X_2 & -\left(\alpha+\frac{r}{k}\right)X_2 & \frac{c_1X_2}{1+aX_2} & 0 \\ (\alpha-r_1)Y_2 & -r_1Y_2 & -c_2Y_2 & -c_2eY_2 \\ 0 & 0 & c_1e_1X_2/(1+aX_2)-c_2e_2Y_2-d_2 & 0 \\ 0 & 0 & 0 & -d_3 \end{pmatrix}$$

The eigenvalues of J_2 are $-d_3$, $c_1e_1X_2/(1+aX_2)-c_2e_2Y_2-d_2$ and the roots of the equation

$$\rho^2+\left(r_1Y_2+\frac{r}{k}X_2\right)\rho+\left[(\alpha-r_1)\left(\alpha+\frac{r}{k}\right)+\frac{rr_1}{k}\right]X_2Y_2=0,$$

The roots of the equation are complex number with negative real part if $\frac{\alpha}{r_1}>1$. So that the point

$E_2(X_2,Y_2,0,0)$ is stable if $\frac{(c_2e_2Y_2+d_2)(1+aX_2)}{c_1e_1X_2}<1$ *and* $\frac{\alpha}{r_1}>1$.

5.4 E_3

The jacobian matrix J_3 of the equilibrium point E_3 is given by

$$J_3 = \begin{pmatrix} n_{11} & n_{12} & n_{13} & 0 \\ 0 & n_{22} & 0 & 0 \\ n_{31} & n_{32} & n_{33} & 0 \\ 0 & 0 & 0 & n_{44} \end{pmatrix}$$

Where $n_{11}=r-\frac{2r}{k}X_3-\frac{c_1Z_3}{(1+aX_3)^2}$, $n_{12}=-\left(\alpha+\frac{r}{k}\right)X_3$, $n_{13}=-\frac{c_1X_3}{1+aX_3}$,

$n_{22}=(\alpha-r_1)X_3-c_2Z_3-d_1$, $n_{31}=\frac{c_1e_1Z_3}{(1+aX_3)^2}$, $n_{32}=c_2e_2Z_3$, $n_{33}=\frac{c_1e_1X_3}{1+aX_3}-d_2$,

$n_{44}=\beta Z_3-d_3$.

The eigenvalues of J_3 are βZ_3-d_3 and $(\alpha-r_1)X_3-c_2Z_3-d_1$ and the roots of the equation

$$\rho^2+\left\{r-\frac{2r}{k}X_3-\frac{c_1Z_3}{(1+aX_3)^2}+\frac{c_1e_1X_3}{1+aX_3}-d_2\right\}\rho$$

$$+\left\{\frac{{c_1}^2e_1Z_3X_3}{(1+aX_3)^3}\right\}+\left\{r-\frac{2r}{k}X_3-\frac{c_1Z_3}{(1+aX_3)^2}\right\}\left(\frac{c_1e_1X_3}{1+aX_3}-d_2\right)=0,$$

If $r-\frac{2r}{k}X_3-\frac{c_1Z_3}{(1+aX_3)^2}>0$, $\frac{c_1e_1X_3}{1+aX_3}-d_2>0$, then the equation have complex roots with

negative real part.

Therefore, the point E_3 is stable if $\beta Z_3 - d_3 < 0, (\alpha - r_1)X_3 - c_2 Z_3 - d_1 < 0,$

$$r - \frac{2r}{k}X_3 - \frac{c_1 Z_3}{(1+aX_3)^2} > 0, \frac{c_1 e_1 X_3}{1+aX_3} - d_2 > 0.$$

5.5 E_4

The jacobian matrix J_4 of the equilibrium point E_4 is given by

$$J_4 = \begin{pmatrix} p_{11} & p_{12} & p_{13} & 0 \\ 0 & p_{22} & 0 & 0 \\ p_{31} & p_{32} & 0 & p_{34} \\ 0 & 0 & p_{43} & 0 \end{pmatrix}$$

Where $p_{11} = r - \frac{2r}{k}X_4 - \frac{c_1 Z_4}{(1+aX_4)^2}$, $p_{12} = -\left(\alpha + \frac{r}{k}\right)X_4$, $p_{13} = -\frac{c_1 X_4}{1+aX_4}$,

$p_{22} = (\alpha - r_1)X_4 - c_2 Z_4 - c_2 e W_4 - d_1$, $p_{31} = \frac{c_1 e_1 Z_4}{(1+aX_4)^2}$, $p_{32} = c_2 e_2 Z_4$, $p_{34} = -\beta Z_4$,

$p_{43} = \beta W_4$.

The eigenvalues of J_4 are $(\alpha - r_1)X_4 - c_2 Z_4 - c_2 e W_4 - d_1$ and the roots of the equation

$$\rho^3 + p_1\rho^2 + p_2\rho + p_3 = 0,$$

where $p_1 = -p_{11} = \frac{2r}{k}X_4 + \frac{c_1 Z_4}{(1+aX_4)^2} - r > 0$, if $\frac{2r}{k}X_4 + \frac{c_1 Z_4}{(1+aX_4)^2} - r > 0,$

$p_2 = -(p_{13}p_{31} + p_{43}p_{34}) = \frac{c_1^2 e_1 Z_4 X_4}{(1+aX_4)^3} + \beta^2 Z_4 W_4 > 0$, $p_3 = p_{11}p_{43}p_{34}$,

$p_1 p_2 - p_3 = p_{11}p_{13}p_{31} > 0$.

So all eigenvalues are of J_4 are negative real parts if $\frac{2r}{k}X_4 + \frac{c_1 Z_4}{(1+aX_4)^2} - r > 0$ and

$(\alpha - r_1)X_4 - c_2 Z_4 - c_2 e W_4 - d_1 < 0.$

Thus, E_4 is stable if $\frac{2r}{k}X_4 + \frac{c_1 Z_4}{(1+aX_4)^2} - r > 0$ and $(\alpha - r_1)X_4 - c_2 Z_4 - c_2 e W_4 - d_1 < 0.$

5.6 E_5

The jacobian matrix J_5 of the equilibrium point E_5 is given by

$$J_5 = \begin{pmatrix} q_{11} & q_{12} & q_{13} & 0 \\ q_{21} & q_{22} & q_{23} & q_{24} \\ q_{31} & q_{32} & 0 & q_{34} \\ 0 & 0 & 0 & q_{44} \end{pmatrix}$$

Where $q_{11} = \frac{c_1 X_5}{1+aX_5}\left(1 - \frac{c_1 X_5}{1+aX_5}\right) - \frac{r}{k}X_5,\ q_{12} = -\left(\alpha + \frac{r}{k}\right)X_5,\ q_{13} = -\frac{c_1 X_5}{1+aX_5},$

$q_{21} = (\alpha - r_1)Y_5,\ q_{22} = r_1 Y_5,\ q_{23} = -c_2 Y_5,\ q_{24} = -c_2 e Y_5,,\ q_{31} = \frac{c_1 e_1 Z_5}{(1+aX_5)^2},$

$q_{32} = c_2 e_2 Z_5,\ q_{34} = -\beta Z_5,\ \ q_{44} = \beta W_5 - d_3.$

The eigenvalues of J_5 are $\beta W_5 - d_3$ and the roots of the equation

$$\rho^3 + q_1\rho^2 + q_2\rho + q_3 = 0,$$

where $q_1 = -\left(q_{11} + q_{22}\right) > 0,$

$q_2 = -q_{31}(q_{12}q_{21} + q_{23}q_{32} + q_{13}q_{31} - q_{11}q_{22}),$

$q_3 = q_{32}(q_{11}q_{23} - q_{13}q_{21}) - q_{31}(q_{12}q_{23} - q_{13}q_{22})$

All the eigenvalues of the above equation have negative real part if $q_1 > 0, q_2 > 0, q_1 q_2 - q_3 > 0.$

Thus, E_5 is stable if $\beta W_5 - d_3 < 0, q_1 > 0, q_2 > 0, q_1 q_2 - q_3 > 0$.

6. STABILITY ANALYSIS OF THE POSITIVE INTERIOR EQUILIBRIUM

Theorem 2. *The positive interior Equilibrium point* $E^*(X^*, Y^*, Z^*, W^*)$ *of the system* (1) is locally asymptotically stable if the following conditions hold

$\sigma_1 > 0, \sigma_4 > 0, \sigma_1\sigma_2 - \sigma_3 > 0, \sigma_3(\sigma_1\sigma_2 - \sigma_3) - \sigma^2{}_1\sigma_4 > 0,$ where $\sigma's$ are given in the proof of the theorem.

Proof: The jacobian matrix J^* of the equilibrium point E^* is given by

$$J^* = \begin{pmatrix} m_{11} & m_{12} & m_{13} & 0 \\ m_{22} & 0 & m_{23} & m_{24} \\ m_{31} & m_{32} & 0 & m_{34} \\ 0 & 0 & m_{43} & 0 \end{pmatrix}$$

where $m_{11} = \frac{c_1 X_5}{1+aX_5}\left(1 - \frac{c_1 X^*}{1+aX^*}\right) - \frac{r}{k}X^*,\ m_{12} = -\left(\alpha + \frac{r}{k}\right)X^*,\ m_{13} = -\frac{c_1 X^*}{1+aX^*},$

$m_{21} = (\alpha - r_1)Y^*,\ \ m_{23} = -c_2 Y^*,\ m_{24} = -c_2 e Y^*,\ \ m_{31} = \frac{c_1 e_1 Z^*}{(1+aZ^*)^2},$

$m_{32} = c_2 e_2 Z^*,\ m_{34} = -\beta Z^*,\ \ m_{43} = \beta W^*.$

The eigenvalues of J^* are the roots of the equation

$$\rho^4 + \sigma_1\rho^3 + \sigma_2\rho^2 + \sigma_3\rho + \sigma_4 = 0,$$

Where

$$\sigma_1 = -(m_{11} + m_{22}),$$
$$\sigma_2 = m_{32}m_{23} + m_{11}m_{22} - m_{12}m_{21} - m_{13}m_{32} - m_{34}m_{43},$$
$$\sigma_3 = m_{11}(m_{32}m_{23} + m_{34}m_{43}) - m_{32}(m_{13}m_{21} + m_{24}m_{43}) + m_{31}(m_{13}m_{22} - m_{12}m_{23}) + m_{22}m_{34}m_{43},$$
$$\sigma_4 = m_{43}\{m_{11}(m_{24}m_{32} - m_{34}m_{22}) + m_{12}(m_{21}m_{34} - m_{24}m_{31})\}.$$

Using the Routh-Hurwitz criteria, the system (1) is locally asymptotically stable about the point E^* if

$$\sigma_1 > 0, \sigma_4 > 0, \sigma_1\sigma_2 - \sigma_3 > 0, \sigma_3(\sigma_1\sigma_2 - \sigma_3) - \sigma^2{}_1\sigma_4 > 0.$$

7. NUMERICAL SIMULATIONS

Parameter	Default Value
r	1.8
k	52
α	0.1
r_1	0.01
c_1	0.3
c_2	0.5
e	0.2
e_1	0.9
a	0.4
e_2	0.7
β	0.09
d_1	0.3
d_2	0.1
d_3	0.25

Table 1: A set of parameter values.

The qualitative behavior of stability of each equilibria of (1) is investigated by using the hypothetical parametric values given in the Table 1.

For the set of parametric values in Table 1 and with initial value $Z_0 = (X_0, Y_0, Z_0, W_0) = (40, 3, 2, 18.5)$, the existence conditions of the coexistence equilibrium point E^* are satis- fied and $E^* = (38.7726, 3.0264, 2.7778, 17.7038)$ is asymptotically stable with eigenvalues $-0.1028 \pm 1.6269i$, $-0.5597 \pm 0.3626i$ (Figure 1(a)). To observe the effects of few param- eters on system (1), firstly, consider $k = 0.4$ and other set of parametric values in Table 1, observe that the predators and infected prey species goes to extinction (Figure 1(b)). Next taking $d_2 = 4.7$ and other set of parametric values in Table 1, the system (1) goes topredators free equilibrium point (Figure 1(c)) is observed. Also, consider $k = 2$, $c_1 = 2$ and other set of parametric

values in Table 1, observe that the predator population goes to ex- tinction (Figure 2(a)). Next taking $d_1 = 2.7$ and other set of parametric values in Table 1, the infected prey goes to extinction (Figure 2(b)) is observed. Next for $d_2 = 0.7$, $d_3 = 0.8$ and other set of parametric values in Table 1, the infected predator goes to extinction (Figure 2(c)) is observed. Lastly, if increase the value of the parameters α from 0.1 to 0.39; β from 0.09 to 0.65; c_2 from 0.7 to 0.8; an oscillation diagrams are plotted as shown in (Figure 4). Finally, the dynamical changes of system (1) due to change the value of

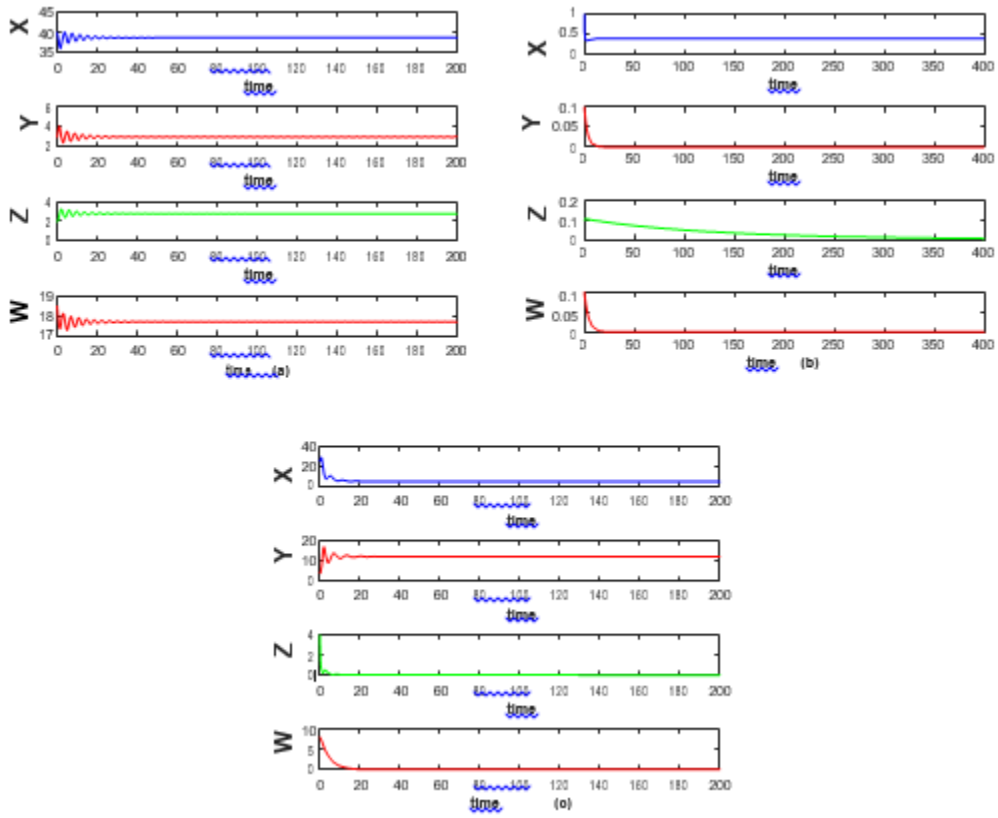


Figure 1: (a) Denotes that all populations coexist in stable position for $r = 1.8$, $r_1 = 0.01$, $k = 52$, $a = 0.4$, $\alpha = 0.1$, $\beta = 0.09$, $c_1 = 0.3$, $c_2 = 0.5$, $e_1 = 0.9$, $e = 0.2$, $e_2 = 0.7$, $d_1 = 0.3$, $d_2 = 0.1$, $d_3 = 0.25$. (b) Denotes that the extinction of infected prey and predators populations for $k = 0.4$ and other parameters fixed as given in (a). (c) Depicts that theextinction of predator species for $d_2 = 4.7$ and other parameters fixed as given in (a).

The parameters α from 0.34 to 0.39; β from 0.35 to 0.65; c_2 from 0.7 to 0.8; a bifurcation diagrams are plotted as shown in (Figure 4).

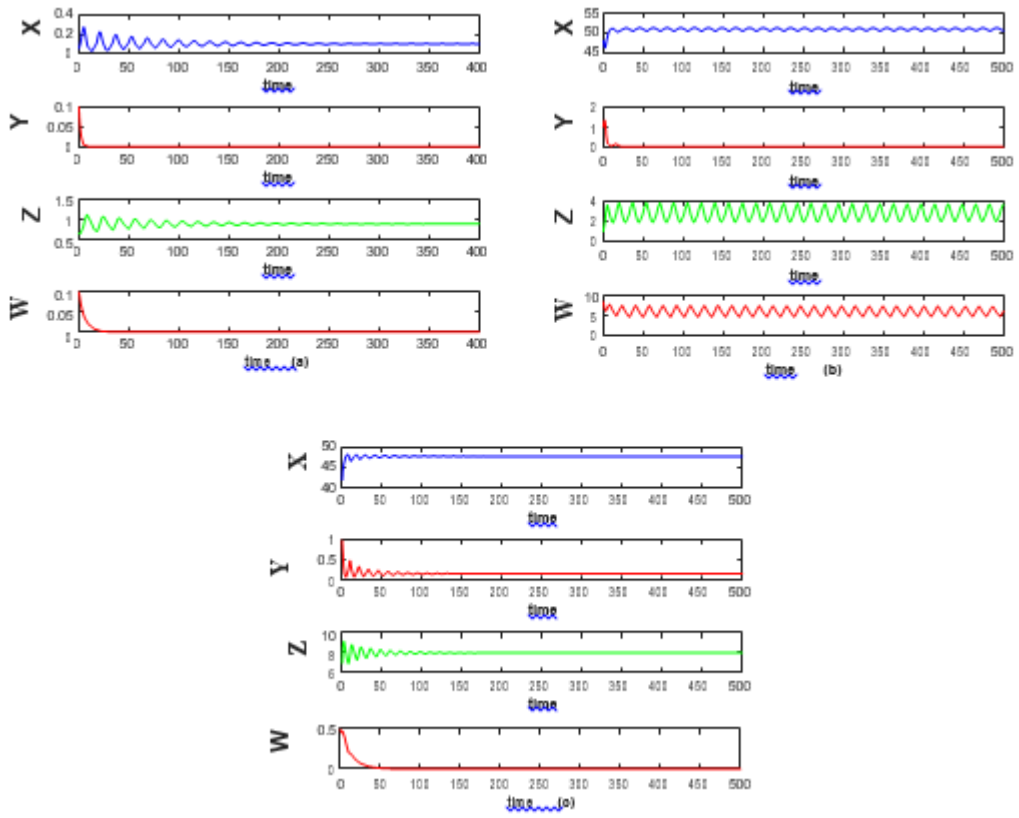


Figure 2: (a) Denotes that the extinction of infected species for for $c_1 = 2, k = 2$ and other parametric values as given in Table 1. (b) Denotes that the infected prey does not persists for $d_1 = 2.7$ and other parameters fixed as given in Table 1. (c) Denotes that the infected predator does not persists for $d_2 = 0.7$, $d_3 = 0.8$ and other parameters fixed as given in Table 1.

8. CONCLUSION

In this paper, a predator-prey model with disease in both populations is proposed. Con- sider that the disease cannot transmit between predator and prey populations. The prey population is subdivided into two groups, one being susceptible prey and another being infected prey. The predator population is also subdivided into two categories, one being susceptible predator and another being infected predator. The system (1) have six equi-librium point. We get five stable boundary equilibrium points corresponding to existence states with different species. A Hopf bifurcation occurs about the interior equilibrium point when the disease transmission rates α, β are increased and predation rate c_2 is increased.

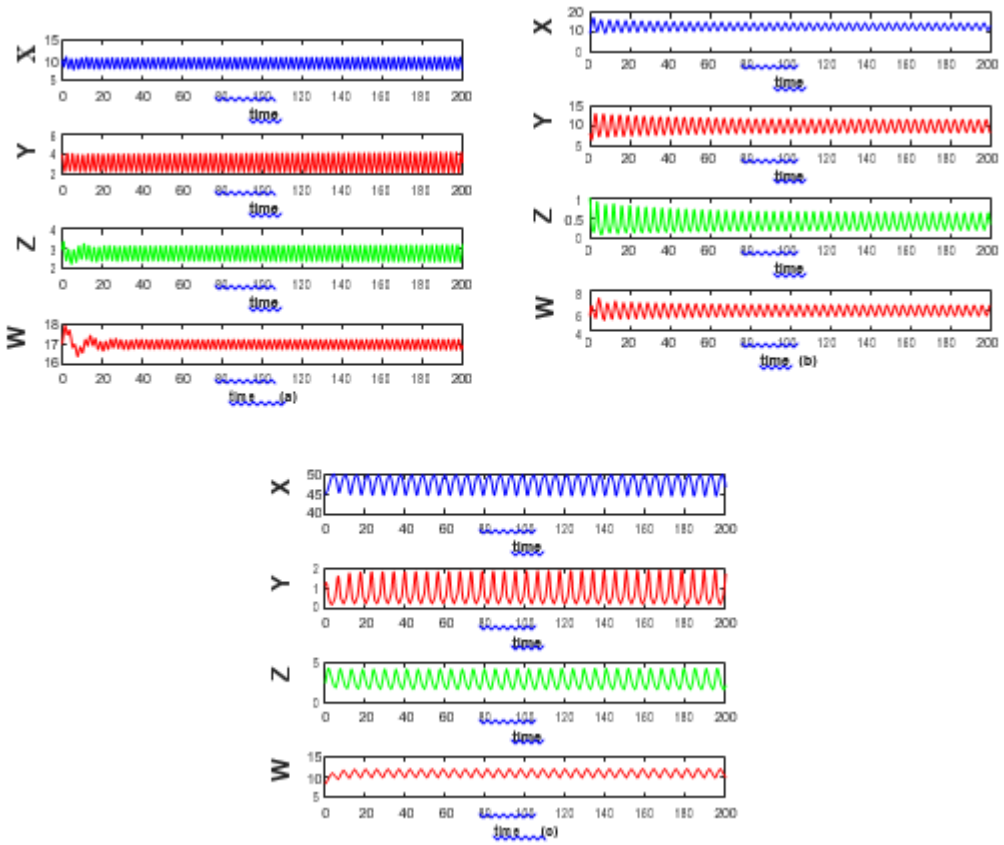
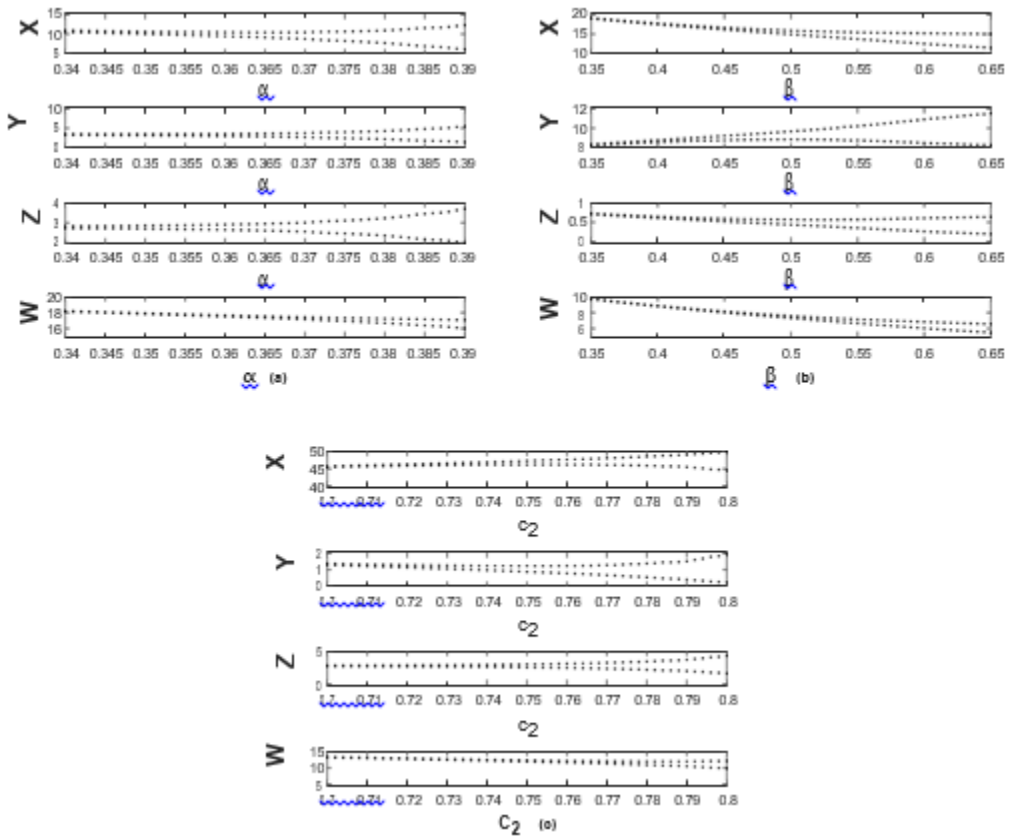


Figure 3: (a) Denotes that the all species coexist in limit cycle oscillation for $\alpha = 0.38$ and other parameters fixed as given in Table 1. (b) Shows that the all populations coexistin limit cycle oscillation for $\beta = 0.65$ and other parametric values as given in Table 1. (c) The figure shows that oscillatory behavior of all species for $c_2 = 0.8$ and other parametersfixed as given in Table 1.



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