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ORIGINAL CONTRIBUTION

Predator-prey dynamics with disease in prey incorporating a prey refuge

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ABSTRACT

A predator-prey model having Holling type three functional response is considered and analyzed in which the disease in the prey population. The disease can only spread over the prey population but not in the predator population is assumed. As a condition prey species has been divided into two groups, one being susceptible prey and another being infected prey. Adding a new feature in this model, in which incorporates an infected prey refuge. It has been discussed the existence of different equilibria and stability analysis at those equilibria. We examined that a Hopf-bifurcation may happen about the interior equilibrium point taking refuge parameter, rate of infection and carrying capacity parameters are bifurcation parameters. The analytical results are supported by numerical simulations.

KEY WORDS: Predator-prey, Eco-epidemiology, Infection, Stability, Hopf-bifurcation

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1. INTRODUCTION

The dynamical relationship between prey and predator is an important part in ecology. To understand the dynamical behavior of such a system is essential mathematical modeling also. On the basis of eating prey the authors [1,2] analyzed co-epidemic models in which the predator species is infected. Some researchers [3, 4, 5] have analysed the prey-predator model, out of which the dynamics of Holling-Tanner model [6] plays a significant part in ecology.

On the other hand use of spatial refuges by the prey is one of the more relevant behavioral traits that affect the dynamics of predator-prey systems. Some academician [7,8,9,10] have investigated the influence of prey refuge and concluded that the stabilizing effect on the prey-predator interaction by using refuge on prey and also use such policy that the prey species can be prevented well from extinction. Some researchers [11, 12, 13, 14] also discussed eco-epidemic model incorporating a prey refuge in predator-prey system. Recently, from the mathematical as well as experimental point of view it is an important issue on the effect of disease on ecological systems. Venturino [15], Haque et al.[16,17],Tewa[18], Hethcote [19], Rahman [20], Chattopadhyay et.all [21,22] discussed the dynamics of prey-predator system with disease in prey population.

Here, a predator prey model with Holling type-III is considered and the inclusion of prey refuge proportional to the infected prey density.Incorporation of infected prey refuge leaves a factor of the infected prey species which are accessible to the predator.

2.Mathematical model

To construct the mathematical model, we make the following assumptions:

Assume, N=the prey population density and z= the predator population density.

In the presence of disease, N=x+y, where x=susceptible class, y=infected class. The disease only transmits from susceptible prey(x) to infected prey (y).

The susceptible prey follows the logistic law:

dx/dt=rx(1-(x+y)/k),

where r = intrinsic birth rate, k= carrying capacity.

The susceptible prey(x) becomes infected by αxy at α (constant rate of infection).

Predator eats susceptible prey by c_1 (x^2 z)/(a+x^2) and infected prey by c_2 (1-m)yz/(a+(1-m)y) at constant rates c_1,c_2 respectively.

'my' is the capacity of a refuge at time t and so refuge protecting 'my' of the infected prey, where 0 < m < 1. This leaves (1 − m)y of the infected prey available to the predator.

Considering the above basic assumptions, we can now write down the following dynamical system:

dx/dt=rx(1-(x+y)/k)-αxy-c_1 (x^2 z)/(a+x^2)≡F_1 (x,y,z),

dy/dt=αxy-c_2 (1-m)yz/(a+(1-m)y)-d_1 y≡F_2 (x,y,z),

dz/(dt)=c_1 e_1 (x^2 z)/(a+x^2)+c_2 e_2 (1-m)yz/(a+(1-m)y)-d_2 z≡F_3 (x,y,z), (1)

with x(0)≥0,y(0)≥0,z(0)≥0.

where e1, e2 are the conversion rates factor for the predator due to consumption of susceptible prey and infected prey respectively; a is the half saturation constants for prey population; d1=death rate of infected prey, d2=death rate of predator.

3. Qualitative analysis of the system

3.1 Boundedness of the System

Theorem1. Every solutions of the system (1) are bounded.

Proof: Let u(t)=x(t)+y(t)+z(t).

Now using the equations (1), we have

du/dt=dx/dt+dy/dt+dz/dt

=rx(1-(x+y)/k)-c_1 (x^2 z)/(a+x^2)-c_2 (1-m)yz/(a+(1-m)y)-d_1 y

+c_1 e_1 (x^2 z)/(a+x^2)+c_2 e_2 (1-m)yz/(a+(1-m)y)-d_2 z

≤rx-r/k x^2-c_1 (1-e_1) (x^2 z)/(a+x^2)-c_2 (1-e_2) (1-m)yz/(a+(1-m)y)-d_1 y-d_2 z

≤rx-r/k x^2-d_1 y-d_2 z,if e_i≤1 for i=1,2.

Therefore,

du/dt+μu≤x(r+μ-r/k x)-(d_1-μ)y-〖(d〗_2-μ)z

du/dt+μu≤k/4r (r+μ-r/k x),if μ=min{d_1,d_2 }

du/dt+μu≤(k(r+μ)^2)/4r.

If 〖 e〗_i<1, for i=1,2 and μ=min{d_1,d_2}, then for each μ≥0 the above inequality becomes

du/dt+μu≤M, where M=(k(r+μ)^2)/4r.

Now by the theory of differential inequality [23] we have,

0≤u(t)≤M/μ (1-e^(-μt))+u(0) e^(-μt)

As t→∞,then 0≤u(t)≤M/μ.

Hence u(t) is bounded. Thus every solutions of the system (1) is bounded.

3.2 Equilibria Analysis

The equilibria E0(0,0,0) and E1(k,0,0) exist for all parametric values.

The predator free equilibrium point E_2 (x_2,y_2,0) exists if R_1=kα/d_1 >1, where x_2=d_1/α,

y_2=(r(kα-d_1))/(α(kα+r)).

The infection free equilibrium point E_3 (x_3,0,z_3) exists if R_2=(c_1 e_1)/d_2 >1, where

x_3=√((ad_2)/(c_1 e_1-d_2)),z_3=r/(kc_1 x_3) (k-x_3)(a+〖x_3〗^2).

The positive interior equilibrium point E^* (x^*,y^*,z^*), where z^*=(αx^*-d_1)[a+(1-m) y^*]/(c_2 (1-m))

y^*=(a^2 d_2+k_1 〖x^*〗^2)/(〖(k〗_(2+) k_3 〖x^*〗^2)(1-m)) and x^* is the positive root of the equation

A_1 z^5+A_2 z^4+A_3 z^3+A_4 z^2+A_5 z+A_6=0, the coefficient are given by

A_1=r/k c_2 (1-m) k_3,A_2=Lc_2 k_1+αc_1 (k_1+αk-3)-rc_2 (1-m) k_3,

A_3=r/k c_2 (1-m)(ak_3+k_2)-c_1 d_1 (k_1+ak_3),

A_4=aLc_2 (k_1+ad_2)+c_1 αa(k_2+ad_2)-rc_2 (1-m)(k_2+ak_3),

A_5=r/k 〖ak_2 c〗_2 (1-m)-c_1 d_1 a(k_2+ad_2),A_6=Lc_2 a^3 d_2-rac_2 k_2 (1-m),

k_1=a(d_2-c_1 e_1),k_2=a(c_2 e_2-d_2),k_3=c_1 e_1+c_2 e_2-d_2,L=r/k+α.

3.3 Stability analysis of equilibria

E_0

The Jacobian matrix J(E_0) of the system (1) about E_0 is

J(E_0)=[■(r&0&0@0&-d_1&0@0&0&-d_2)]

So, the eigenvalues of J(E_0) are r, -d_1and 〖-d〗_2. So E_0 is unstable because one eigenvalue is positive.

E_1

The Jacobian matrix J(E_1) of the system (1) about E_1 is

J(E_1)=[■(-r&-(r/k+α)k&-c_1 k^2/(a+k^2)@0&αk-d_1&0@0&0&c_1 e_1 k^2/(a+k^2)-d_2)]

So, the eigenvalues of J(E_1)are -r, αk-d_1and 〖 c〗_1 e_1 k^2/(a+k^2)-d_2. So 〖 E〗_1isstable if αk/d_1 <1 and

(c_1 e_1 k^2)/(a+k^2)<1.

E_2

The Jacobian matrixJ(E_2) of the system (1) about E_2 is

J(E_2)=[■(-r/k x_2&-(r/k+α) x_2&-c_1 〖x_2〗^2/(a+ 〖x_2〗^2)@0&αx_2-d_1&(〖-c〗_2 (1-m)y_2)/(a+(1-m)y_2)@0&0&k_11)]

Where k_11=c_1 e_1 k^2/(a+k^2)+c_2 e_2 ((1-m)y_2)/(a+(1-m)y_2)-d_2. The eigenvalues of J(E_2) are 〖 c〗_1 e_1 k^2/(a+k^2)+c_2 e_2 ((1-m)y_2)/(a+(1-m)y_2)-d_2 and the roots of λ^2+(rd_1)/kα λ+rd_1 (1-(2d_1)/kα)=0. So, 〖 E〗_2 is stable if c_1 e_1 k^2/(a+k^2)+c_2 e_2 ((1-m)y_2)/(a+(1-m)y_2)-d_2<0 and (2d_1)/kα<1.

E_3

The Jacobian matrix J(E_3) of the system (1) aboutE_3 is

J(E_3)=[■(q_11&q_12&q_13@0&q_22&0@q_31&q_32&0)]

Where q_11=r-2 r/k x_3-2c_1 a (x_3 z_3)/(a+x_3^2)^2 ,q_12=-(r/k+α) x_3,q_13=-c_1 (x_3^2)/(a+x_3^2),

q_22=αx_3-c_2 (1-m) z_3-d_1,q_31= 2c_1 a (x_3 z_3)/(a+x_3^2)^2 ,q_32= c_2 e_2 (1-m) z_3.

Eigenvalues of J(E_3) are αx_3-c_2 (1-m) z_3-d_1 and the roots of

λ^2+{2 r/k x_3+2c_1 a (x_3 z_3)/(a+x_3^2)^2 -r}λ+rd_1 (1-(2d_1)/kα)=0.

Hence E_(3)is stable if x_3-c_2 (1-m) z_3-d_1<0 and 2 r/k x_3+2c_1 a (x_3 z_3)/(a+x_3^2)^2 -r>0.

Theorem 2. The positive interior equilibrium point E^* (x^*,y^*,z^*) is locally asymptotically stable if β_1>0,β_2>0,β_1 β_2-β_3>0 , where β_i's are given in the proof of the theorem.

Proof: The jacobian matrix of the system (1) around E^* is

J(E^*)=[■(-m_11&-m_12&-m_13@m_21&m_22&-m_23@m_31&m_32&-m_33)]

Where m_11={r/k x^*+(2c_1 x^* z^*)/(a+x^*)^2 }-c_1 〖x^*〗^2/(a+ 〖x^*〗^2) z^*>0,m_12=(r/k+α) x^*>0,m_13=c_1 〖x^*〗^2/(a+ 〖x^*〗^2)>0,

m_21=αy^*>0,m_22=αx^*-d_1-(c_2 a(1-m) z^*)/{a+(1-m) y^* }^2 >0,m_23=(c_2 a(1-m) y^*)/(a+(1-m) y^*)>0,

m_31=(c_1 e_1 x^* z^*)/(a+ 〖x^*〗^2)^2 >0,m_32=(2c_2 e_2 a(1-m) z^*)/(a+(1-m) y^*)>0,m_33=0

Eigenvalues of J(E^*) at E^* are the roots of ρ^3+β_1 ρ^2+β_2 ρ+β_3=0,

where

β_1=(m_11-m_22+m_33)

β_2=m_33 (m_11-m_22)+m_13 m_31+m_12 m_21+m_23 m_32-m_22 m_11

〖 β〗_3=m_12 (m_33 m_21-m_23 m_31)+m_13 〖(m〗_21 m_32+m_31 m_22)+m_11 (m_23 m_32-m_22 m_33)

〖β_1 β_2- β〗_3= 〖(m〗_11-m_22){m_12 m_21+m_11 (m_33-m_22)}+m_13 (m_11 m_31-m_33 m_32)

+(m_33-m_22){m_23 m_32+m_33 (m_11-m_22)} 〖+m_32 (m〗_12 m_23-m_21 m_13)

By the Routh-Hurwitz criteria, all roots of ρ^3+β_1 ρ^2+β_2 ρ+β_3=0, have negative real root parts if and only if β_1>0,β_2>0,β_1 β_2-β_3>0. Therefore, E^* is locally asymptotically stable if β_1>0,β_2>0,β_1 β_2-β_3>0.

4. Numerical simulations

Parameter	r	a	k	α	c_1	c_2	e_1	e_2	m	d_1	d_2
Defalt Value	2	5	26	0.1	0.3	0.7	0.3	0.57	0.2	0.3	0.2

Table1: A set of parameter values.

Occurrence and termination of disease studied here numerically. The existence conditions of the coexistence equilibrium point E∗ is satisfied in a set of parametric values shownin Table1 and E∗= (18.5144, 1.9446, 18.1620) with initial value Z0 = (19, 2.5, 17) is stable with eigenvalues -0.09033±i .5519, −0.3849 (see Figure 1(a)). Next to observe the effects of some parameters on system (1), we first consider k = 3 and r = 2, α = 0.1, a = 5, c1 =0.3, c2= 0.7, e1= 0.3, e2= 0.67, d1= 0.3, d2 = 0.2, m = 0.2, observe that the infected preys and predators goes to extinction (see Figure 1(b)). Next, we consider d2 = 0.3 andr=2,α=0.1,a=5,c1=0.3,c2=0.7,e1=0.3,e2=0.67,d1=0.3,k=26,m=0.2, observe that the predator goes to extinction (see Figure 1(c)). Next, we consider c1 = 1.4 and r=2 ,α=0.1, a=5, c2=0.7, e1=0.3,e2=0.67,d1=0.3,d2=0.2,m=0.2,k=26, observe that the infected prey goes to extinction (seeFigure1(d)). When value of the parameters m is increased from 0.2 to .66 and other parameters are fixed, it is observed that the solution of (1) changes from stable to oscillatory behavior (see Fig 2 (a)). Again, when α = 0.1416 and other parameters are fixed, it is observed that the solution of (1) changes from stable to oscillatory behavior (see Figure 2(b)). Next, when k=33 and other parameters are fixed, it is observed that the solution of (1) changes from stable to oscillatory behavior (see Figure 2(d)).

Finally, the dynamical changes of system (1)due to change of value of the parameters k (from 27 to 33), α (from 0.1 to 0.12, from 0.06 to 0.1 and from 0.065 to 0.13), m (from 0.2 to 0.66) bifurcation diagrams are plotted in Figures 3(a), 3(b), 3(c), 3(d), and 3(e) respectively.

5. Conclusions:

An eco-epidemiological predator-prey model has been partially applied in this work.Through the analysis an eco-epidemiological predator-prey model has been developedwhere only prey is affected by an infectious disease.There are five equilibrium points,namely E0, E1, E2, E3 and E∗. Existence and stability analysis in each equilibrium points are established under some conditions.

It has been observed that the great influence of this infected prey refuge on each population. Susceptible prey and predator density both are decreased for increasing the amount of infected prey refuge. On the other hand infected prey density is increased for increasing the amount of infected prey refuge. The interior equilibrium E^* of (1) becomes stable when the infected prey density is increased. So the refuge parameter (m) is an important determinant of stability of the population. The stability switching and a Hopf-bifurcation may occur at E∗ taking m, k and α are bifurcation parameters. The analytical results cannot be completed without numerical observations. So all important analytical results are numerically verified using Matlab here.

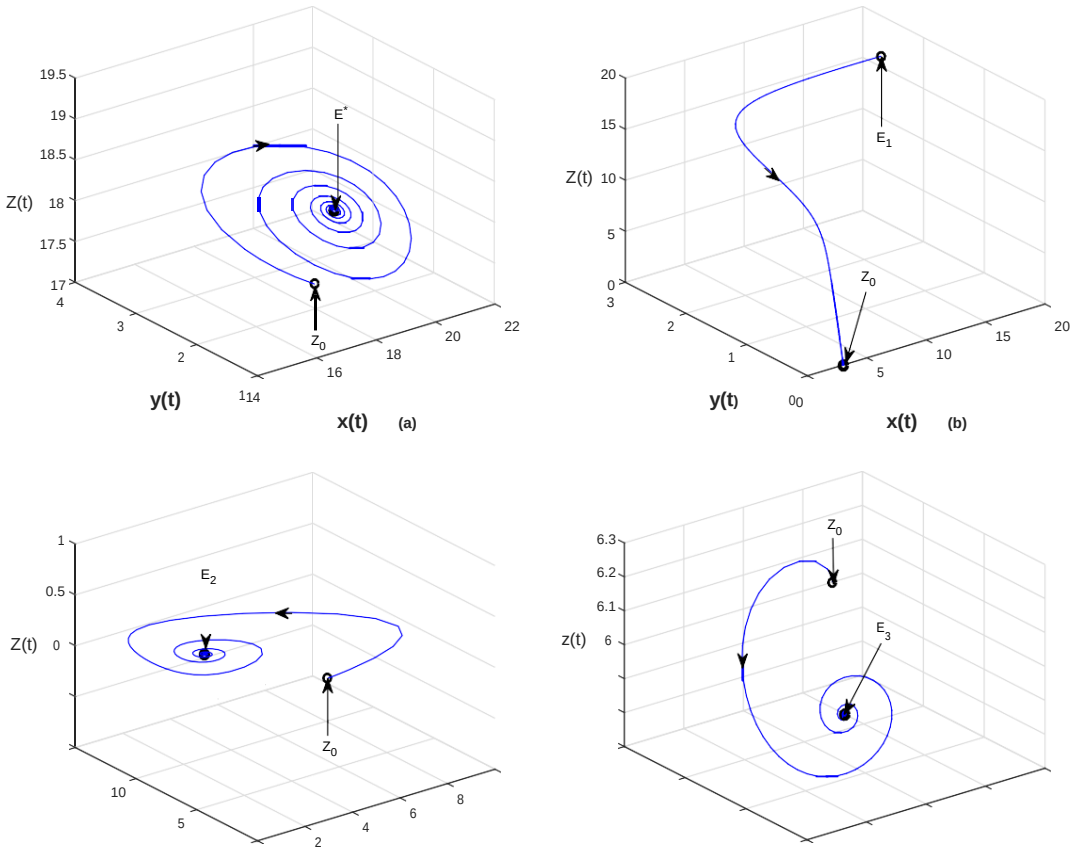


Figure1: The figure (a) denotes E∗ is stable for the set of parameter r = 2, a = 5, k = 27, α = 0.1, c1=0.3,c2=0.7,e1=0.3,e2=0.67,d1=0.3,d2=0.2,m=0.2; Figure(b) depicts that E1 is stable fork=3 with other set of parameters in Fig.1(a); Figure (c) depicts that E2is stable for d2= 0.3 with other setof parameters in Fig.1(a); Figure (d) depicts that E3is stable for c1= 1.4 with other set of parameters in Fig.1(a).

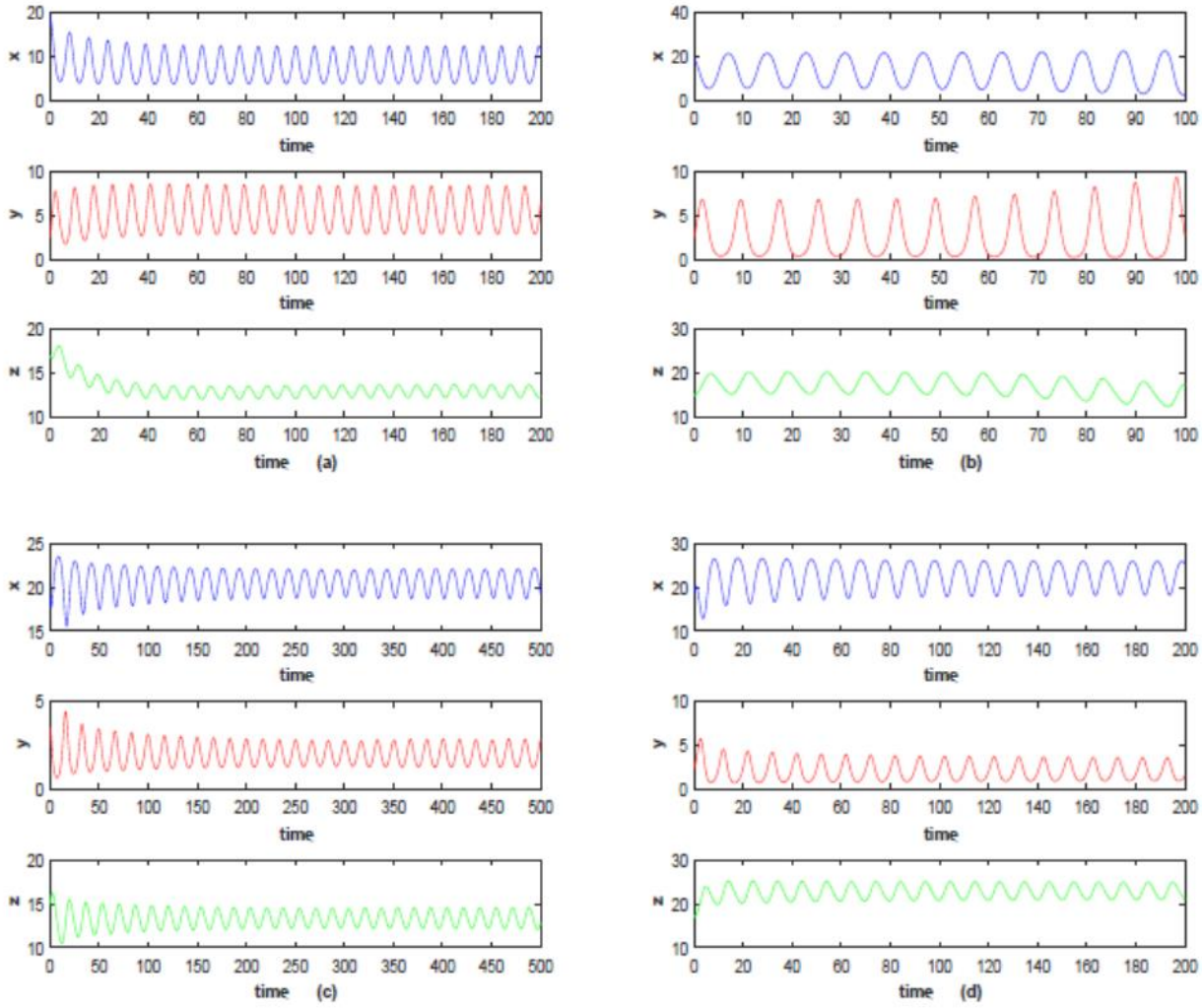


Figure 2: Figures (a), (b), (c), (d) denote oscillatory behavior of all species for the parameters α, k, m respectively.

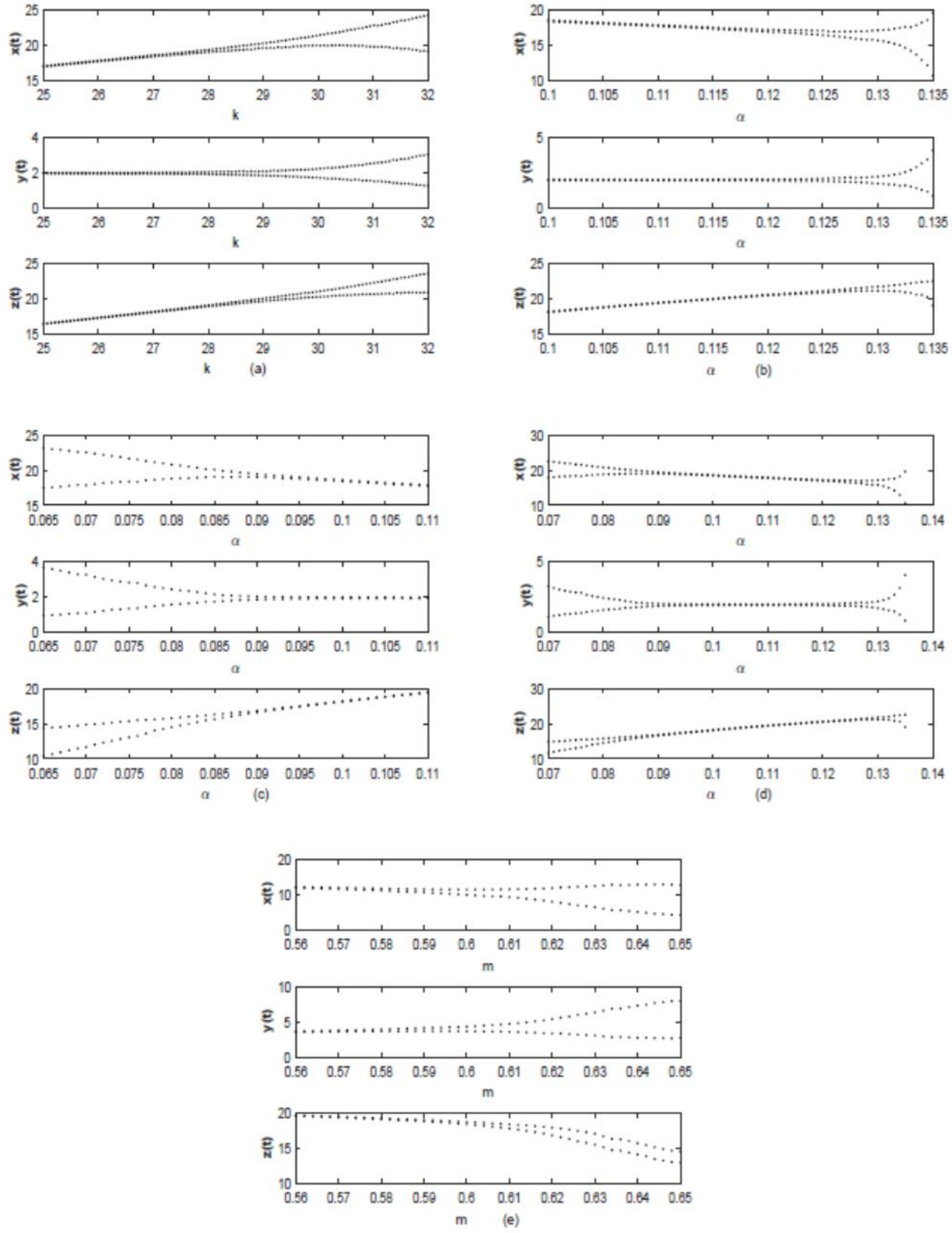


Figure 3: Figure (a) denotes the bifurcation diagram for k; Figures (b), (c) and (d) denote the bifurcation diagrams for α; Figure (e) denotes the bifurcation diagram for m.

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